Function Notation

If you went to an accountant and asked how much you'd have to pay in taxes this year, the accountant would probably respond by asking, "How much did you make last year?" He can't answer your question until he has some numbers from you to start his calculation. Once he knows those numbers, he'll do the same math that he's done for many other clients to figure out how much you owe in taxes.

Functions are like this. For any particular function, you give it a number, it performs some calculations using that number, and it gives you a result. It doesn't matter what the number is at the start, the calculations will be the same.

One of the nice things about functions is we don't have to know what the calculated result is for us to talk about it. The tax accountant can talk to you about "the amount of taxes you'll have to pay on your income" even if he doesn't know the dollar value yet. He's describing the answer by telling you which set of calculations he's going to do (the ones on the tax form) and the number he's going to start with (your income).

In math, we like to label things with symbols and letters. In the same way that we can represent a number by using a variable like x, we name functions with letters and indicate what numbers they operate on. The symbol for a function usually looks like $f(\ )$. Inside the parentheses, we put the number or expression that we are going to apply the function to. (This is called the argument of the function.) When we're defining what calculations the function performs, we use a variable like x. The function that adds 3 to any number would look like this:

$$f(x) = x + 3$$

We can then refer to the result when we apply this calculation to any number or expression. If we want to write, "the answer we get when we add three to nine" we can write $f(9)$. We would read these symbols as "f of nine".

You might be wondering why we bother with this notation when we could easily write "9 + 3". As you progress through mathematics, you'll encounter functions that are more complicated than this one, where the easiest way to refer to a result is that it's the answer you get when you perform a particular calculations on a number. An example of this that you're already familiar with is fractions.

The answer to the problem 3 ÷ 7 is a repeating decimal. The length of the repeating part is six digits. To write this number down exactly it would be tedious to have to memorize these digits. We'd have to do that for all the other answers to division problems that don't work out evenly. Instead we refer to "the answer you get when you divide 3 by 7" as a fraction: $\frac{3}{7}$. Fraction form tells us what numbers we use in the calculation and what operation to apply to them, even if it doesn't convey its value. Fractions are a form of function notation on two numbers. Change the numbers and you change the value.

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