Polynomial Division Worksheet

For each of the following examples find polynomials $Q(x)$ and $R(x)$ such that

$$\frac{f(x)}{g(x)} = Q(x) + \frac{R(x)}{g(x)}$$

and $\deg R < \deg g$.

Ex. Let

$$f(x) = 2x^3 - 5x^2 + 4x - 7 \quad g(x) = 2x^2 - 3x + 1.$$

$$\begin{aligned}
\frac{f(x)}{g(x)} = \frac{2x^3 - 5x^2 + 4x - 7}{2x^2 - 3x + 1} &= \frac{2x^3 - 3x^2 + x + 3x^2 - x - 5x^2 + 4x - 7}{2x^2 - 3x + 1} \\
&= \frac{2x^3 - 3x^2 + x}{2x^2 - 3x + 1} + \frac{3x^2 - x - 5x^2 + 4x - 7}{2x^2 - 3x + 1} \\
&= \frac{x(2x^2 - 3x^1 + 1)}{2x^2 - 3x + 1} + \frac{-2x^2 + 3x - 7}{2x^2 - 3x + 1} \\
&= x + \frac{-2x^2 + 3x - 1 - 3x + 1 + 3x - 7}{2x^2 - 3x + 1} \\
&= x + \frac{-2x^2 + 3x - 1}{2x^2 - 3x + 1} + \frac{-3x + 1 + 3x - 7}{2x^2 - 3x + 1} \\
&= x + \frac{-1(2x^2 - 3x + 1)}{2x^2 - 3x + 1} + \frac{-6}{2x^2 - 3x + 1} \\
&= \boxed{x - 1 + \frac{-6}{2x^2 - 3x + 1}}
\end{aligned}$$

1. $f(x) = 2x^2 - 5x + 1$ and $g(x) = x + 1$.
2. $f(x) = -3x^3 + 2x + 1$ and $g(x) = x - 3$
3. $f(x) = 2x^2 - 6x + 7$ and $g(x) = 2x + 1$
4. $f(x) = x^6 + 1$ and $g(x) = x^2 + 1$

For each of the following, find all of the roots of f and factor $f(x)$ as much as possible.

5. $f(x) = x^2 + 5x - 7$
6. $f(x) = -2x^3 + 9x^2 - 12x + 4$ given that $f(2) = 0$.
7. $f(x) = 2x^3 + 9x^2 + 11x + 2$, given that $f(-2) = 0$.
8. $f(x) = x^4 + 6x^2 - 40$